Machine Learning Techniques for Value-at-Risk (VaR) calculation

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Agenda

- VaR Calculation
  - Parametric
  - Non-Parametric
  - Semi-Parametric
- Shortcomings of established VaR models
- VaR Estimation leveraging ML/DL
- Back-testing / Evaluation Metrics
- Univariate Analysis
- Multivariate Analysis
- Infinitech Architecture
- Next steps
- Questions

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VaR Calculation

- The Value-at-Risk for a given time horizon *t* and confidence level *α* is the loss in market value over the time horizon *t* that is exceeded with probability *P*.
  - $P(r_t \leq VaR_{\alpha,t}) = 1\text{-}\alpha$
- For instance, in case of a daily VaR and a confidence level 99% (i.e. $t = 1,\ a = 0.99$) for an examined period of 1000 days, a valid VaR model should produce :
  - No more than 10 exceedances
  - No clusters of exceedances

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VaR Calculation: Parametric Method

- Portfolio returns and their distribution should be theoretically defined prior to VaR estimation.
- Most of the parametric models are simple in terms of implementation assuming Gaussian or Student-t distribution. However, the above distributions do not apply to the most of the financial time-series.
- Some well-known methods of the parametric category are the Variance-Covariance Method (VC) and many GARCH-variants Methods, such as the Risk Metrics model.
- For example, when VC Method is applied assuming Gaussian distribution, the VaR can be easily obtained using the below equation, provided that the variance-covariance matrix Σ of returns has been calculated.

$$VaR^a = z_{1-a} * \sqrt{W^T \sum W}$$

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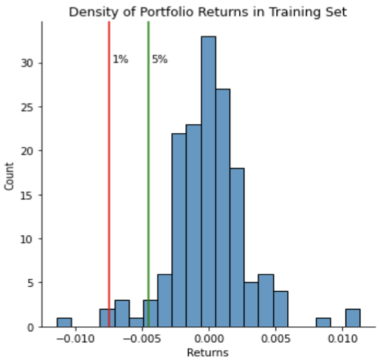
VaR Calculation: Non-Parametric Method

- Assumptions regarding the distribution of returns are not required.
- The main advantage of this type of method is the low computational complexity. However, this method fails to capture unseen fluctuations that are not present in the utilized history-window
- The Historical Simulation (HS) is the main representative of this method, where the empirical distribution of past portfolio returns is used to calculate VaR. The VaR can be easily obtained for a given history-window by taking the required quantile of the empirical distribution .



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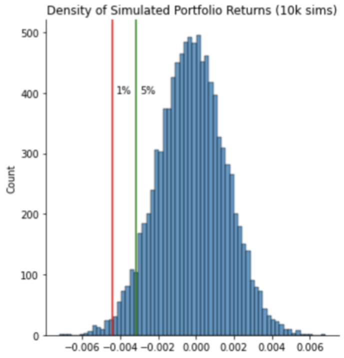
VaR Calculation: Semi-Parametric Method

- Some assumptions regarding the distribution of returns are made either for the "error" distribution or for the extremes of the distribution or for the model dynamics.
- The Monte Carlo Method is the main semi-parametric method which generates random scenarios for future portfolio returns, drawing their distribution based on some non-linear pricing models. MC is more reliable when dealing with complex portfolios and complicated risk factors. MC core assumption is that the risk factor has a known probability distribution i.e. that market factors follow certain stochastic processes which are used to estimate future returns.
- VaR at the target confidence level α (i.e. 99%) is derived by taking the 1-α (i.e. 0.01) quantile of the generated distribution.



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Shortcomings of established VaR models

The major challenges that most of the existing VaR methods cannot address are:

- Severe VaR violations, where the portfolio realized a loss exceeding the VaR value when high market downturns occur, due to dependency between the VaR predictions, especially for 99% confidence level.
- High excess loss, beyond the VaR threshold, when high market downturns occur, due to the fat tails of the financial time-series distribution and the leverage effect.
- The opportunity cost of capital, where firms would unnecessarily reserve according to the VaR estimates of their portfolios rarely is taken into account.

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VaR Estimation leveraging ML/DL (1/2)

- Our model based on the DeepAR algorithm[1], which utilizes RNNs to produce probabilistic forecasts in the form of Monte Carlo samples that can be used to compute consistent quantile estimates in a certain prediction horizon.
- This algorithm can be fed simultaneously with several similar time series, enabling cross-learning between them.
- Those features make DeepAR an ideal candidate model to predict VaR of FX portfolios, where the individual time series share similar dependencies and the overall goal is to draw the portfolio's returns distribution.

1. D. Salinas, V. Flunkert, J. Gasthaus, and T. Januschowski, Deepar: Probabilistic forecasting with autoregressive recurrent networks," International Journal of Forecasting, vol. 36, no. 3, pp. 1181–1191, 2020.

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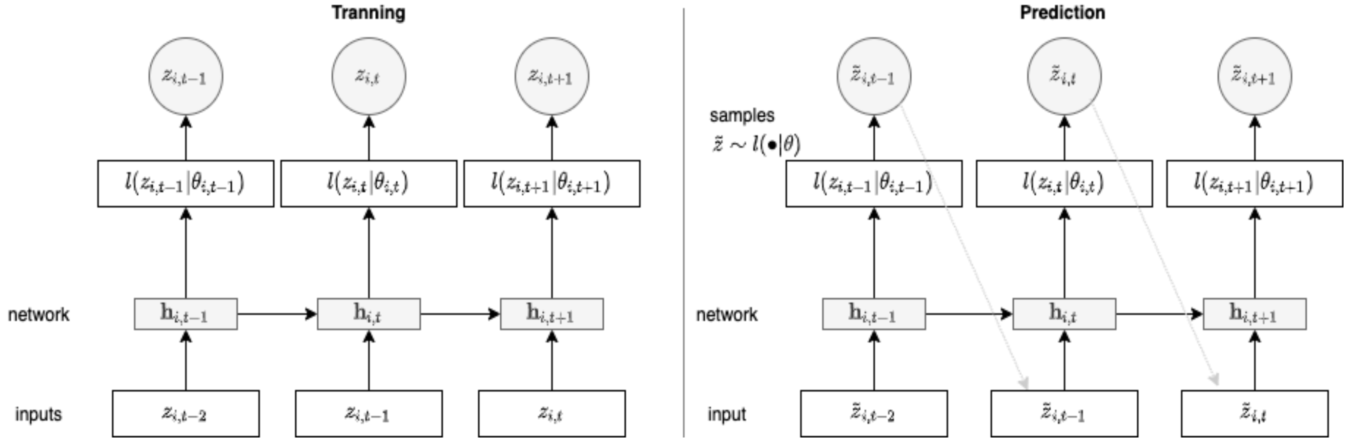
VaR Estimation leveraging ML/DL (2/2)

- DeepAR models the conditional distribution $P(z_{i,t0:T}|z_{i,1:t0-1})$ of the time series z_i for future time-step from t_0 to T given the past values of the z_i from time-step 1 to t_0-1. It is assumed that the model distribution $Q(z_i;t_0{:}T\ |z_i;1{:}t_0\text{-}1)$ consists of a product of likelihood factors.
- Several samples can be easily generated from the estimated returns distribution, in a MC fashion. Then VaR can be easily obtained.



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Back-testing / Evaluation Metrics (1\2)

- Hit Variable (I_t): $$I_t = \begin{cases} 1, & \text{if } VaR_t > PnL_t \\ 0, & \text{otherwise} \end{cases} \qquad N_{violations} = \sum_{t=1}^{T} I_t$$
- Expected Violations (E[v]) : $E[v] = (1-\alpha) * N_{days}$
- Violation Rate: $$r_{Violations} = \frac{N_{Violations}}{N_{days}}$$
- Quadratic Loss (l_{QL}): $$l_{QL} = \sum_{t=1}^{N} I_t(1 + (PnL_t - VaR_t)^2)$$
- Smooth Loss (l_Q): $$l_Q = \frac{1}{N}\sum_{t=1}^{N}(\alpha - (1+e^{dm})^{-1})m \text{ , where } d = 25,\ m = PnL_t - VaR_t^{\alpha}$$
- Tick Loss (l_T): $$l_T = \sum_{t=1}^{N}(\alpha - I_t)(PnL_t - VaR_t^{\alpha})$$
- Firm Loss (l_F): $$l_F = \begin{cases} (PnL_t - VaR_t)^2, & \text{if } PnL_t < VaR_t \\ -aVaR_t, & \text{otherwise} \end{cases}$$

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Back-testing / Evaluation Metrics (2\2)

VaR forecasts are valid if and only if the violation process I_t satisfies the following two assumptions:

- The unconditional coverage (UC) hypothesis: the unconditional probability of a violation must be equal to the α coverage rate.
- The independence (IND) hypothesis: VaR violations observed at two different dates must be independently distributed.

Validity of these assumptions was tested by exploiting both the Christoffersen's conditional coverage test and the Dynamic Quantile (DQ) test proposed by Engle and Manganelli (2004) .

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Univariate Analysis (1/4) - AUDUSD

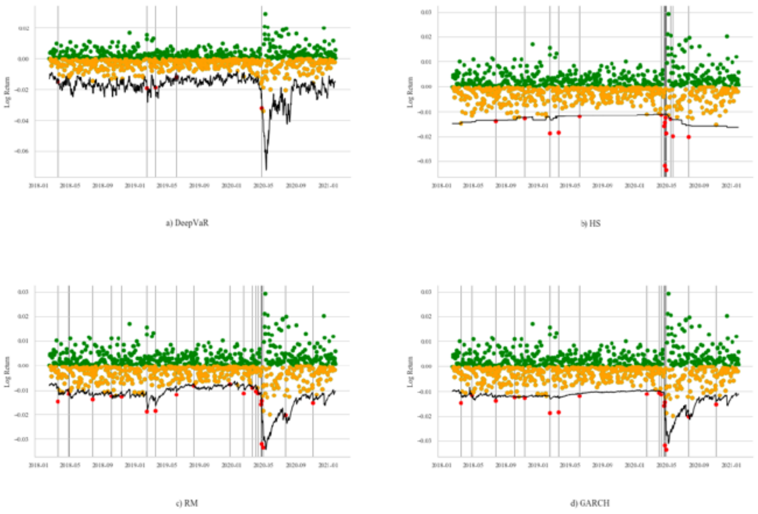

Fig. 2: AUDUSD: $VaR^{99\%}$ performance per model.

Model	$E[v]$	v	r_v	l_{QL}	l_Q	l_T	l_F
DeepVaR	9.28	**5**	**0.005388**	**0.005388**	**-0.006318**	**0.000196**	**0.023342**
HS	9.28	15	0.016164	0.016165	-0.005187	0.000224	0.029225
RM	9.28	21	0.022629	0.022630	-0.004715	0.000205	0.034574
GARCH	9.28	17	0.018319	0.018320	-0.004879	0.000200	0.030592

TABLE I: Performance of $VaR^{99\%}$ models in AUDUSD series.

Model	LR_{uc}	LR_{ind}	LR_{cc}	DQ
DeepVaR	2.386 [0.122]	0.054 [0.816]	2.441 [0.295]	2.158 [0.905]
HS	3.014 [0.083]	10.682** [0.001]	13.696** [0.001]	124.125** [0.0]
RM	11.036** [0.001]	2.931 [0.087]	13.966** [0.001]	31.907** [0.0]
GARCH	5.224* [0.022]	1.013 [0.314]	6.237* [0.044]	17.616** [0.007]

TABLE II: Coverage and independence tests of $VaR^{99\%}$ models in AUDUSD series. In brackets are the p-values.

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Univariate Analysis (2/4) - GBPUSD

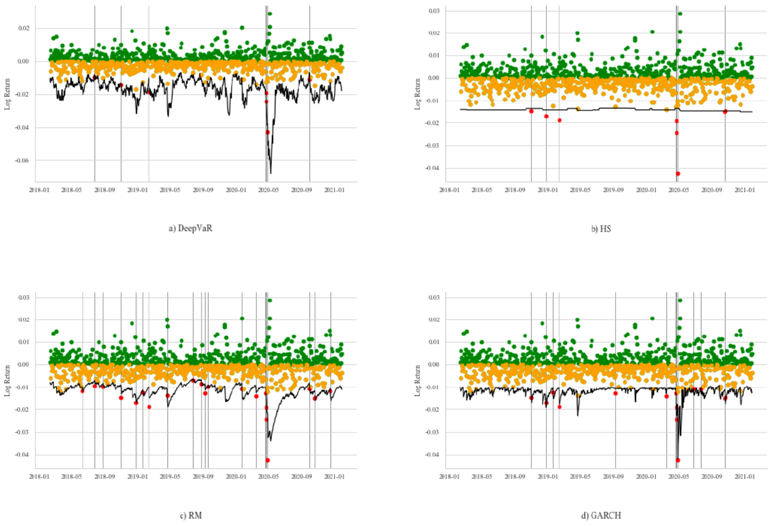

Fig. 3: GBPUSD: $VaR^{99\%}$ performance per model.

Model	$E[v]$	v	r_v	l_{QL}	l_Q	l_T	l_F
DeepVaR	9.28	**7**	**0.007543**	**0.007543**	**-0.006169**	0.000192	0.024539
HS	9.28	8	0.008621	0.008622	-0.005572	0.000201	**0.022799**
RM	9.28	22	0.023707	0.023708	-0.004674	0.000212	0.035301
GARCH	9.28	14	0.015086	0.015087	-0.004848	**0.000190**	0.027096

TABLE III: Performance of $VaR^{99\%}$ models in GBPUSD series.

Model	LR_{uc}	LR_{ind}	LR_{cc}	DQ
DeepVaR	0.613 [0.434]	4.258* [0.039]	4.871 [0.088]	27.815** [0.0]
HS	0.184 [0.668]	3.713 [0.054]	3.897 [0.142]	31.297** [0.0]
RM	12.745** [0.0]	0.366 [0.545]	13.11** [0.001]	36.725** [0.0]
GARCH	2.108 [0.147]	1.623 [0.203]	3.731 [0.155]	35.0** [0.0]

TABLE IV: Coverage and independence tests of $VaR^{99\%}$ models in GBPUSD series. In brackets are the p-values.

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Univariate Analysis (3/4) - USDJPY

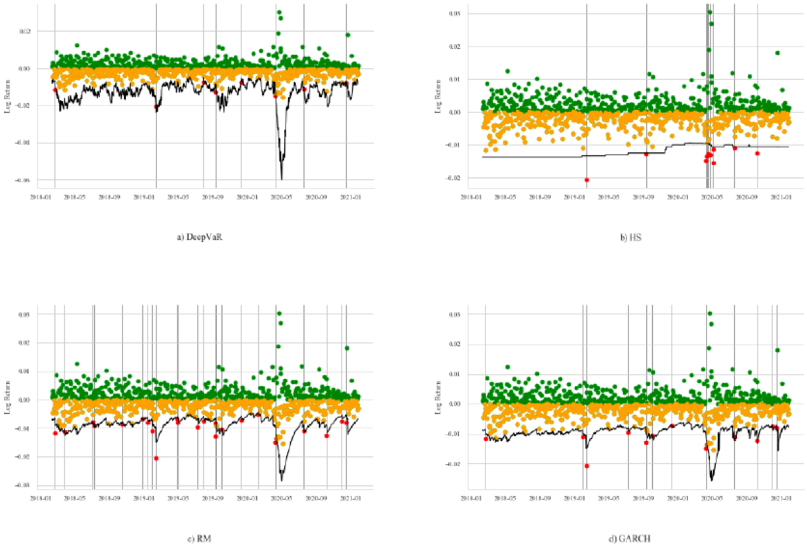

Fig. 4: USDJPY: $VaR^{99\%}$ performance per model.

Model	$E[v]$	v	r_v	l_{QL}	l_Q	l_T	l_F
DeepVaR	9.28	**8**	**0.008621**	**0.008621**	**-0.004931**	0.000151	**0.021463**
HS	9.28	11	0.011853	0.011854	-0.004890	0.000158	0.023872
RM	9.28	24	0.025862	0.025862	-0.003659	0.000150	0.034562
GARCH	9.28	13	0.014009	0.014009	-0.003962	**0.000131**	0.023506

TABLE V: Performance of $VaR^{99\%}$ models in USDJPY series.

Model	LR_{uc}	LR_{ind}	LR_{cc}	DQ
DeepVaR	0.184 [0.668]	0.139 [0.709]	0.324 [0.851]	2.857 [0.827]
HS	0.308 [0.579]	2.477 [0.116]	2.785 [0.248]	49.276** [0.0]
RM	16.439** [0.0]	0.207 [0.649]	16.646** [0.0]	36.755** [0.0]
GARCH	1.347 [0.246]	0.37 [0.543]	1.717 [0.424]	8.448 [0.207]

TABLE VI: Coverage and independence tests of $VaR^{99\%}$ models in USDJPY series. In brackets are the p-values.

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Univariate Analysis (4/4) - EURUSD

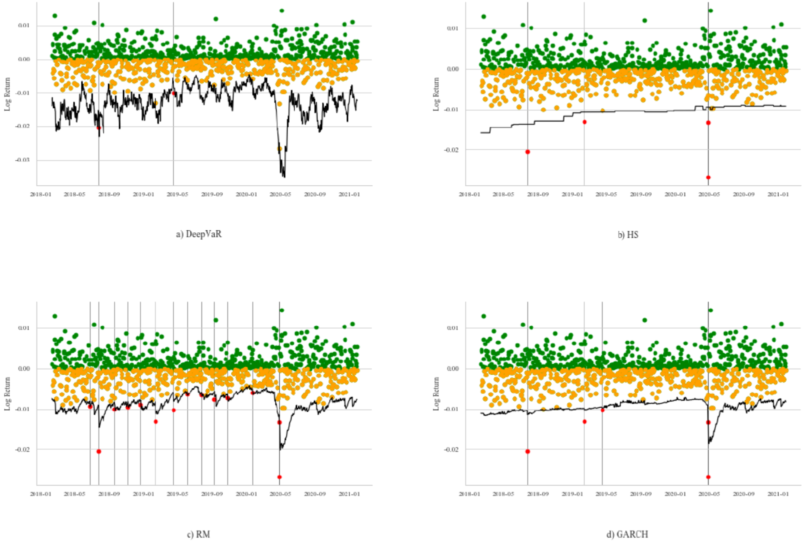

Fig. 5: EURUSD: $VaR^{99\%}$ performance per model.

Model	$E[v]$	v	r_v	l_{QL}	l_Q	l_T	l_F
DeepVaR	9.28	**2**	**0.002155**	**0.002155**	**-0.005080**	0.000138	0.015008
HS	9.28	4	0.004310	0.004311	-0.004585	0.000144	0.015391
RM	9.28	14	0.015086	0.015087	-0.003624	0.000133	0.023539
GARCH	9.28	5	0.005388	0.005388	-0.004038	**0.000131**	**0.014940**

TABLE VII: Performance of $VaR^{99\%}$ models in EURUSD series.

Model	LR_{uc}	LR_{ind}	LR_{cc}	DQ
DeepVaR	8.463** [0.004]	0.009 [0.926]	8.472* [0.014]	5.786 [0.448]
HS	3.846 [0.05]	0.035 [0.852]	3.881 [0.144]	29.349** [0.0]
RM	2.108 [0.147]	0.429 [0.512]	2.537 [0.281]	9.598 [0.143]
GARCH	2.386 [0.122]	0.054 [0.816]	2.441 [0.295]	22.326** [0.001]

TABLE VIII: Coverage and independence tests of $VaR^{99\%}$ models in EURUSD series. In brackets are the p-values.

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Multivariate Analysis (1/2)

- Model performance was tested in 1000 random portfolios. These portfolios were created randomly by producing both positive and negative positions on the aforementioned FX assets.
- To compute the VaR of a portfolio, the correlation among the FX instruments should be taken into account. In this case, the VaR of a portfolio for a given day can be estimated by:

$$VaR_p^\alpha = \sqrt{VRV^T}$$

where V is a vector of the weighted VaR estimates per instrument, $V = [w_1 VaR_1^\alpha, w_2 VaR_2^\alpha, w_3 VaR_3^\alpha, w_4 VaR_4^\alpha]$.

and R is the correlation matrix of FX assets' daily returns

$$R = \begin{pmatrix} 1 & \rho_{1,2} & \rho_{1,2} & \rho_{1,4} \\ \rho_{2,1} & 1 & \rho_{2,3} & \rho_{2,4} \\ \rho_{3,1} & \rho_{3,2} & 1 & \rho_{3,4} \\ \rho_{4,1} & \rho_{4,2} & \rho_{4,3} & 1 \end{pmatrix}$$

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Multivariate Analysis (2/2)

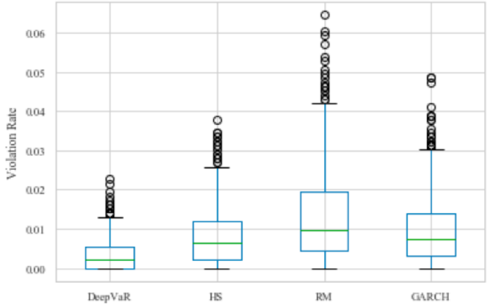

Fig. 6: Violation Rate of $VaR^{99\%}$ models

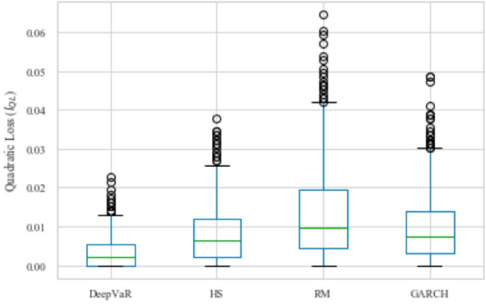

Fig. 7: Quadratic Loss of $VaR^{99\%}$ models

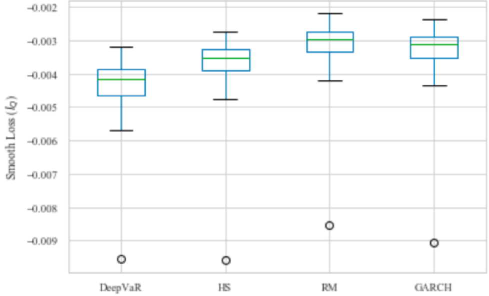

Fig. 8: Smooth Loss of $VaR^{99\%}$ models

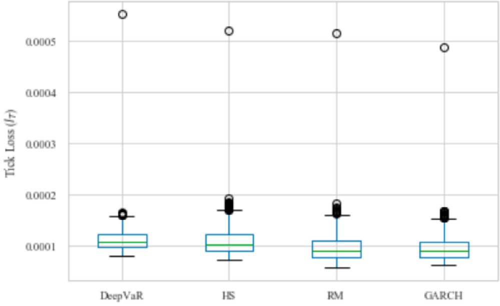

Fig. 9: Tick Loss of $VaR^{99\%}$ models

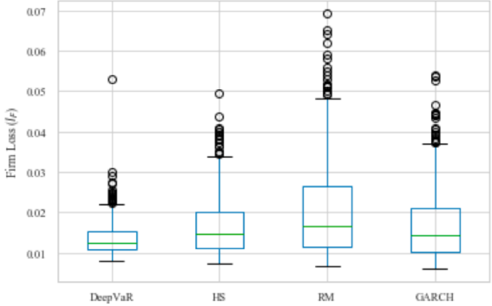

Fig. 10: Firm Loss of $VaR^{99\%}$ models

Model	$E[v]$	v	r_v	l_{QL}	l_Q	l_T	l_F
DeepVaR	9.28	**2.903097**	**0.003128**	**0.003128**	**-0.004273**	0.000110	**0.013512**
HS	9.28	7.278721	0.007843	0.007844	-0.003609	0.000107	0.016179
RM	9.28	12.135864	0.013077	0.013078	-0.003048	0.000096	0.020018
GARCH	9.28	8.610390	0.009278	0.009279	-0.003209	**0.000095**	0.016602

TABLE IX: Average performance of $VaR^{99\%}$ models over the FX portfolios.

Model	LR_{uc}	LR_{ind}	LR_{cc}	DQ
DeepVaR	72.8	**95.5**	**80.6**	**84.6**
HS	**76.9**	72.0	59.5	36.7
RM	65.2	95.3	68.3	55.9
GARCH	76.5	92.9	77.5	63.0

TABLE X: Percentage of portfolios passed the coverage and independence tests of $VaR^{99\%}$ per model in significant level 95%

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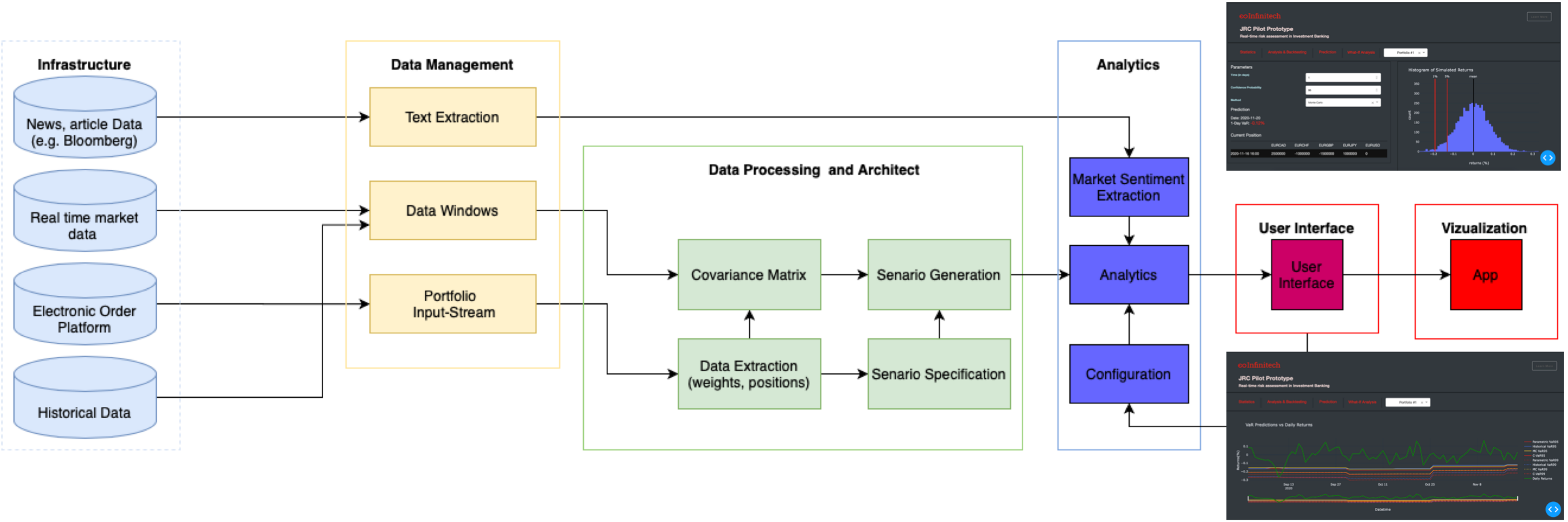
Infinitech Architecture (1/2)



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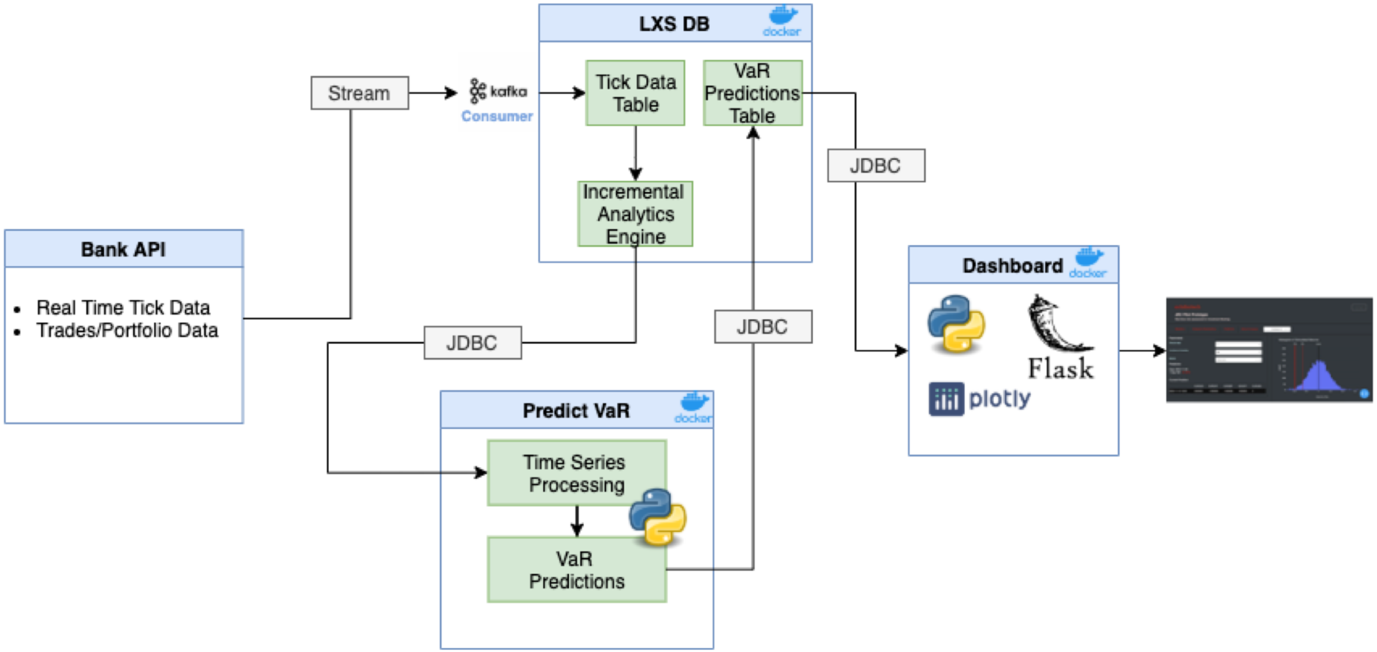
Infinitech Architecture (2/2)



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Next steps

- Improve the efficiency of the proposed DeepVaR approach in terms of high frequency trading.
  - Required time for model training should be optimized.
- Other sources of complementary information could be integrated for improved results, such as sentiment analysis of tweets and news.

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Thank you!

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